

For a graph H , its homomorphism density in graphs naturally extends to the space of two-variable symmetric functions W in L^p , $p \geq e(H)$, denoted by $t_H(W)$. One may then define corresponding functionals $\|W\|_H := |t_H(W)|^{1/e(H)}$ and $\|W\|_{r(H)} := t_H(|W|)^{1/e(H)}$ and say that H is (semi-)norming if $\|\cdot\|_H$ is a (semi-)norm and that H is weakly norming if $\|\cdot\|_{r(H)}$ is a norm.

We obtain some results that contribute to the theory of (weakly) norming graphs. Firstly, we show that ‘twisted’ blow-ups of cycles, which include $K_{5,5} \setminus C_{10}$ and $C_6 \square K_2$, are not weakly norming. This answers two questions of Hatami, who asked whether the two graphs are weakly norming. Secondly, we prove that $\|\cdot\|_{r(H)}$ is not uniformly convex nor uniformly smooth, provided that H is weakly norming. This answers another question of Hatami, who estimated the modulus of convexity and smoothness of $\|\cdot\|_H$. We also prove that every graph H without isolated vertices is (weakly) norming if and only if each component is an isomorphic copy of a (weakly) norming graph. This strong factorisation result allows us to assume connectivity of H when studying graph norms. Based on joint work with Frederik Garbe, Jan Hladký, and Bjarne Schülke.