The Radiative Uniqueness Conjecture for Bubbling Wave Maps

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Abstract. One of the most fundamental questions in partial differential equations is that of regularity and the possible breakdown of solutions. We will discuss this question for solutions to a canonical example of a geometric wave equation; energy critical wave maps. Breakthrough works of Krieger-Schlag-Tataru, Rodnianski-Sterbenz and Raphaël-Rodnianski produced examples of wave maps that develop singularities in finite time. These solutions break down by concentrating energy at a point in space (via *bubbling* a harmonic map) but have a regular limit, away from the singular point, as time approaches the final time of existence. The regular limit is referred to as the *radiation.* This mechanism of breakdown occurs in many other PDE including energy critical wave equations, Schrödinger maps and Yang-Mills equations. A basic question is the following:

- Can we give a precise description of *all* bubbling singularities for wave maps with the goal of finding the natural unique continuation of such solutions past the singularity?

In this talk, we will discuss recent work (joint with J. Jendrej and A. Lawrie) which is the first to directly and explicitly connect the radiative component to the bubbling dynamics by constructing and classifying bubbling solutions with a simple form of prescribed radiation. Our results serve as an important first step in formulating and proving the following *Radiative Uniqueness Conjecture* for a large class of wave maps: every bubbling solution is uniquely characterized by it's radiation, and thus, every bubbling solution can be uniquely continued past blow-up time while conserving energy.