The exact format of the final exam can be found on the Math Department web page. The actual final exam will consist of 16 multiple-choice problems, 1 graph analysis problem, and 7 show-all-work problems.

These sample problems are meant only to help you prepare for the final exam; they do not consist of a complete study guide. While all core topics are covered in these sample problems, not all types of problems appear in this list. Your main and complete study guide consists of the official textbook problems and MathXL problems.

1. For each part, find \( f'(x) \) as a function of \( x \) only. Do not simplify your answer.
   
   (a) \( f(x) = (x^3 + x)^{10} \)  
   (b) \( f(x) = x^{\sin(2x)} \)

2. Find each antiderivative or integral.
   
   (a) \( \int \frac{2x + \sqrt{x} - 1}{x} \, dx \)  
   (b) \( \int (2x + 3)^{1/2} \, dx \)  
   (c) \( \int_0^1 e^x (1 + e^{-2x}) \, dx \)  
   (d) \( \int_0^{\pi/2} (1 + \sin(x))^5 \cos(x) \, dx \)

3. If \( x^3 + xy + y^2 = 7 \), find \( \frac{dy}{dx} \) at \( (1, 2) \).

4. Calculate each limit.
   
   (a) \( \lim_{x \to 0} \left( \frac{\sin(x)^2}{\sin(2x^2)} \right) \)  
   (b) \( \lim_{x \to 1} \left( \frac{\ln(x^2 + 2) - \ln(3)}{x - 1} \right) \)

5. Find the largest possible area of a rectangle whose base lies on the \( x \)-axis and whose upper vertices lie on the parabola \( y = 6 - x^2 \).

6. A car traveling north at 40 mi/hr and a truck traveling east at 30 mi/hr leave an intersection at the same time. At what rate will the distance between them be changing 4 hours later?

7. Find the absolute minimum and maximum of \( f(x) = (6x + 1)e^{3x} \) on the interval \([-1000, 1000]\).

8. The marginal revenue of a certain product is \( R'(x) = -9x^2 + 17x + 30 \), where \( x \) is the level of production. Assume \( R(0) = 0 \). Find the market price that maximizes revenue.

9. The parts of this question are not related.
   
   (a) Find \( F'(x) \) if \( F(x) = \int_{-1}^{x} \frac{t^5}{3 + t^6} \, dt \).
   
   (b) Find \( \int_0^5 f(t) \, dt \) if \( f(x) = \begin{cases} 
   x & , \quad x < 1 \\
   1 & , \quad x \geq 1 
\end{cases} \).

10. Use linear approximation or differentials to estimate \( (33.6)^{1/5} \).
11. Consider the function \( f \) and its derivatives below.

\[
f(x) = \frac{(x-1)^2}{(x+2)(x-4)} \quad f'(x) = \frac{-18(x-1)}{(x+2)^2(x-4)^2} \quad f''(x) = \frac{54((x-1)^2+3)}{(x+2)^3(x-4)^3}
\]

Find the vertical and horizontal asymptotes of \( f \). Then find where \( f \) is decreasing, where \( f \) is increasing, where \( f \) is concave down, and where \( f \) is concave up. Calculate the \( x \)-coordinates of all local minima, local maxima, and points of inflection.

12. Let \( f(x) = \frac{x+2}{x-3} \). Use the limit definition of derivative to find \( f'(2) \).

13. Estimate the area under the graph of \( f(x) = x^2 + 5x \) from \( x = 0 \) to \( x = 4 \) using a Riemann sum with right endpoints and 4 rectangles. Simplify your answer.

14. A radioactive frog hops out of a pond full of nuclear waste. If its level of radioactivity declines to \( 1/3 \) of its original value in 30 days, when will its level of radioactivity reach \( 1/100 \) of its original value?

\[\text{Hint: Use the exponential growth formula } P(t) = P_0 e^{rt}.\]

15. For each part, calculate the limit or show it does not exist.

\[
\begin{align*}
\text{(a) } & \lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x} \\
\text{(b) } & \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^{3x} \\
\text{(c) } & \lim_{x \to 0} \left( \frac{\sin(5x) - 5x}{x^3} \right)
\end{align*}
\]

16. Find the derivative of each function.

\[
\begin{align*}
\text{(a) } f(x) &= \tan(3x^2 + e) \\
\text{(b) } f(x) &= e^{x/(x+1)}
\end{align*}
\]

17. Find each antiderivative or integral.

\[
\begin{align*}
\text{(a) } & \int t^2 \cos(1 - t^3) \, dt \\
\text{(c) } & \int_2^3 \frac{\ln(x)}{x} \, dx \\
\text{(b) } & \int \sqrt{x-1} \, dx \\
\text{(d) } & \int_0^{\ln(3)} e^{2x} \sqrt{e^{2x} - 1} \, dx
\end{align*}
\]

18. Find the equation of the line normal to the curve \( 5x^2y + 2y^3 = 22 \) at the point (2, 1).

19. Find the absolute minimum and absolute maximum values of \( f(x) = x^3 - 12x + 5 \) on \([-5, 3]\).

20. The marginal cost (in dollars) of a certain product is \( C'(x) = 6x^2 + 30x + 200 \). If it costs $250 to produce 1 unit, how much does it cost to produce 10 units?

21. An open cylindrical can (without top) must have a volume of \( 16\pi \) cm\(^3\). The cost of the bottom is 2$/cm^2$ and the cost of the curved surface is 1$/cm^2$. Find the radius and height of the least expensive can. Justify that your answer does, in fact, give the minimum cost.

\[\text{Hint: The volume of a cylinder is } \pi r^2 h. \text{ The surface area of the curved surface is } 2\pi rh, \text{ and the surface area of the top or bottom is } \pi r^2.\]

22. Use linear approximation or differentials to estimate \( \sqrt{78} \).
23. The altitude of a triangle is increasing at a rate of 1 ft/min. while the area is increasing at a rate of 2 ft/min. At what rate is the base of the triangle changing when the altitude is 10 ft. and the area is 100 ft$^2$?

24. Consider the function $f$ and its derivatives below.

$$f(x) = \frac{24}{x^3 + 8} \quad , \quad f'(x) = \frac{-72x^2}{(x^3 + 8)^2} \quad , \quad f''(x) = \frac{288x(x^3 - 4)}{(x^3 + 8)^3}$$

Find the vertical and horizontal asymptotes of $f$. Then find where $f$ is decreasing, where $f$ is increasing, where $f$ is concave down, and where $f$ is concave up. Calculate the $x$-coordinates of all local minima, local maxima, and points of inflection.

25. For what values of $a$ and $b$ is the following function continuous for all $x$?

$$g(x) = \begin{cases} 
ax + 2b & , \ x \leq 0 \\
x^2 + 3a - b & , \ 0 < x \leq 2 \\
3x - 5 & , \ x > 2
\end{cases}$$

26. Let $f(x) = \frac{2e^x + 3}{1 - e^x}$.

(a) Find all horizontal asymptotes of $f$, if any.

(b) Find all vertical asymptotes of $f$. Then at each vertical asymptote, find both corresponding one-sided limits.