1. Simplify the expression \( \frac{|2-x|}{x-2} \) if \( x > 2 \).

**Solution**

If \( x > 2 \), then \( 2-x < 0 \), and so \( |2-x| = -(2-x) = x-2 \). Hence \( \frac{|2-x|}{x-2} = \frac{x-2}{x-2} = 1 \).

2. Find all solutions to the equation \( 2^{x^2-2x} = 8 \).

**Solution**

The equation is equivalent to \( 2^{x^2-2x} = 2^3 \), or \( x^2 - 2x = 3 \). After some algebra we have \((x-3)(x+1) = 0\), and so the solutions are \( x = -1 \) and \( x = 3 \).

3. Find the domain of \( f(x) = \frac{\ln(x)}{x-2} \). Write your answer in interval notation.

**Solution**

Note that the domain of \( \ln(x) \) is \((0, \infty)\). Hence the domain of \( f \) is \((0, 2) \cup (2, \infty)\) (the value \( x = 2 \) must be excluded since \( f(x) \) is undefined for \( x = 2 \) due to division by 0).

4. Solve the inequality \( \frac{3x+6}{x(x-4)} \leq 0 \). Write your answer in interval notation.

**Solution**

Use the cut-point (or sign chart) method. For our sign chart, the cut points are found by setting the numerator and denominator to 0 separately. Hence the cut points are \( x = -2 \), \( x = 0 \), and \( x = 4 \). Now we test the truth of the inequality using one point from each corresponding subinterval.

<table>
<thead>
<tr>
<th>interval</th>
<th>test point</th>
<th>sign of ( \frac{3x+6}{x(x-4)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -2))</td>
<td>( x = -3 )</td>
<td>( \oplus ) = ( \ominus )</td>
</tr>
<tr>
<td>((-2, 0))</td>
<td>( x = -1 )</td>
<td>( \oplus ) = ( \ominus )</td>
</tr>
<tr>
<td>((0, 4))</td>
<td>( x = 1 )</td>
<td>( \oplus ) = ( \ominus )</td>
</tr>
<tr>
<td>((4, \infty))</td>
<td>( x = 5 )</td>
<td>( \oplus ) = ( \ominus )</td>
</tr>
</tbody>
</table>

Checking the cut points themselves, we see the inequality is satisfied at \( x = -2 \) but neither \( x = 0 \) nor \( x = 4 \). So the final answer is: \((-\infty, -2] \cup (0, 4)\).
5. An account in a certain bank pays 5% annual interest, compounded continuously. An initial deposit of $200 is made into the account. How many years does it take for the $200 to double? **You must write an exact answer in terms of logarithms.**

**Solution**
The value of the account $t$ years after the initial deposit is $P(t) = 200e^{0.05t}$. The time taken to double in value is the time $T$ such that $P(T) = 400$. Solving the equation $200e^{0.05T} = 400$ gives $T = \ln(2)/0.05 = 20 \ln(2)$ years.

6. For each part, calculate the limit or show that it does not exist.

(a) $\lim_{x \to 0} \left( \frac{\sin(5x)}{3x} \cos(4x) \right)$

(b) $\lim_{x \to -2} \left( \frac{x^2 + 3x + 2}{x^2 + x - 2} \right)$

(c) $\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right)$

**Solution**

(a) Recall that $\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1$ for any $a \neq 0$. Hence we have

$$\lim_{x \to 0} \left( \frac{\sin(5x)}{3x} \cos(4x) \right) = \lim_{x \to 0} \left( \frac{5}{3} \cdot \frac{\sin(5x)}{5x} \cdot \cos(4x) \right) = \frac{5}{3} \cdot 1 \cdot 1 = \frac{5}{3}$$

(b) Cancel common factors, and then use direct substitution.

$$\lim_{x \to -2} \left( \frac{x^2 + 3x + 2}{x^2 + x - 2} \right) = \lim_{x \to -2} \left( \frac{(x + 2)(x + 1)}{(x + 2)(x - 1)} \right) = \lim_{x \to -2} \left( \frac{x + 1}{x - 1} \right) = \frac{-2 + 1}{-2 - 1} = \frac{-1}{3}$$

(c) Find a common denominator, cancel common factors, and then use direct substitution.

$$\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right) = \lim_{x \to 0} \left( \frac{x + 1 - 1}{x^2 + x} \right) = \lim_{x \to 0} \left( \frac{1}{x + 1} \right) = \frac{1}{0 + 1} = 1$$

7. For each part, calculate $f'(x)$. Do not simplify your answer after computing the derivative.

(a) $f(x) = \frac{\tan(x)}{\pi - \sec(x)}$

(b) $f(x) = \cos(e^{-3x})$

(c) $f(x) = \sqrt{\ln(x^2 + 4) + x \sin(2x)}$

(d) $f(x) = \frac{e^{1/x}}{x^{2/3} + x^{1/3}}$

**Solution**

(a) Use quotient rule.

$$f'(x) = \frac{(\pi - \sec x)(\sec^2 x) - (\tan x)(-\sec x \tan x)}{(\pi - \sec x)^2}$$

(b) Use the chain rule twice.

$$f'(x) = -\sin(e^{-3x}) \cdot e^{-3x} \cdot (-3)$$
(c) Use chain rule first on the outermost square root function. Then use chain rule and product rule to compute the derivative of the inner function.

\[ f'(x) = \frac{1}{2} \left( \ln(x^2 + 4) + x \sin(2x) \right)^{-1/2} \cdot \left( \frac{2x}{x^2 + 4} + \sin(2x) + 2x \cos(2x) \right) \]

(d) Start with quotient rule. When differentiating the numerator, use chain rule.

\[ f'(x) = \frac{e^{1/x} \cdot \left( -\frac{1}{x^2} \right) \cdot (x^{2/3} + x^{1/3}) - e^{1/x} \cdot \left( \frac{2}{3}x^{-1/3} + \frac{1}{3}x^{-2/3} \right)}{(x^{2/3} + x^{1/3})^2} \]

8. The graph of \( f(x) \) is given below. Find all values of \( x \) in the interval \((-4, 4)\) for which \( f \) is not continuous.

![Graph of f(x)](image)

**Solution**

The function \( f(x) \) is discontinuous at \( x = -3, x = -1, x = 1, \) and \( x = 2. \)

9. Some values of \( g, h, g', \) and \( h' \) are given below. Use this table to answer parts (a) and (b).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
<th>( h(x) )</th>
<th>( h'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>-9</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-1</td>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

(a) Let \( f(x) = 3g(x)h(x) \). Calculate \( f'(2) \).
(b) Let \( F(x) = g(\sqrt{x}) \). Calculate \( F'(4) \).

**Solution**

(a) Use product rule.

\[ f'(x) = 3g'(x)h(x) + 3g(x)h'(x) \]

Then substitute \( x = 2 \) and use the table of values.

\[ f'(2) = 3(-9)(1) + 3(-3)(5) = -72 \]
(b) Use chain rule.

\[ F'(x) = g'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \]

Then substitute \( x = 4 \) and use the table of values.

\[ F'(4) = g'(2) \cdot \frac{1}{2 \cdot 2} = -\frac{9}{4} \]

10. Find an equation of the line normal to the graph of \( f(x) = 2x^2 - \ln(x) + 3 \) at \( x = 1 \). (Recall that the normal line is perpendicular to the tangent line.)

**Solution**
The derivative at a general point is

\[ f'(x) = 4x - \frac{1}{x} \]

Hence \( f'(1) = 3 \), and so the slope of the normal line is \(-1/3\). The normal line must pass through \((1, f(1)) = (1, 5)\). Hence the equation of the normal line is

\[ y - 5 = -\frac{1}{3}(x - 1) \]

11. Let \( f(x) = 3\sqrt{x} \). Use the limit definition of the derivative to find \( f'(x) \). Show all work.

**Solution**
We have the following work.

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
  &= \lim_{h \to 0} \frac{3\sqrt{x + h} - 3\sqrt{x}}{h} \\
  &= \lim_{h \to 0} \frac{9(x + h) - 9x}{h(3\sqrt{x + h} + 3\sqrt{x})} \\
  &= \lim_{h \to 0} \frac{9h}{h(3\sqrt{x + h} + 3\sqrt{x})} \\
  &= \lim_{h \to 0} \frac{9}{3\sqrt{x + h} + 3\sqrt{x}} = \frac{3}{2\sqrt{x}}
\end{align*}
\]

12. Find the values of the constants \( a \) and \( b \) that make \( f \) continuous at \( x = 9 \).

\[
f(x) = \begin{cases} 
  \sin(2\pi x) - 2ax, & x < 9 \\
  b, & x = 9 \\
  \frac{x - 9}{\sqrt{x} - 3}, & x > 9 
\end{cases}
\]

You must use proper calculus and notation to give a complete and clear justification for your answer.
Solution
If \( f \) is to be continuous at \( x = 9 \), then the left-limit, right-limit, and function value must all be equal at \( x = 9 \). So we first calculate each of these values.

\[
\lim_{x \to 9^-} f(x) = \lim_{x \to 9^-} (\sin(2\pi x) - 2ax) = \sin(18\pi) - 18a = -18a
\]

\[
\lim_{x \to 9^+} f(x) = \lim_{x \to 9^+} \left( \frac{x - 9}{\sqrt{x} - 3} \right) = \lim_{x \to 9^+} (\sqrt{x} + 3) = 6
\]

\[
f(9) = b
\]

These three values must be equal, so that \(-18a = 6 = b\), whence \( a = -\frac{1}{3} \) and \( b = 6 \).

13. Find the \( x \)-coordinate of each point on the graph of \( y = \frac{1}{\sqrt{x}}(x^3 + 15) \) where the tangent line is perpendicular to the line \( x + 5y = 1 \).

Solution
The slope of the given line \( x + 5y = 1 \) is \(-\frac{1}{5}\), whence we want to find all tangent lines with slope 5. Let \( f(x) = \frac{1}{\sqrt{x}}(x^3 + 15) \). So we must solve the equation \( f'(x) = 5 \). First we calculate \( f'(x) \) by rewriting \( f(x) \) in terms of power functions and using power rule.

\[
f(x) = x^{5/2} + 15x^{-1/2} \implies f'(x) = \frac{5}{2}x^{3/2} - \frac{15}{2}x^{-3/2}
\]

Now we set up and solve the equation \( f'(x) = 5 \).

\[
\frac{5}{2}x^{3/2} - \frac{15}{2}x^{-3/2} = 5
\]

\[
5x^{3/2} - 15x^{-3/2} = 10
\]

\[
x^{3/2} - 3x^{-3/2} = 2
\]

\[
x^3 - 3 = 2x^{3/2}
\]

\[
x^3 - 2x^{3/2} - 3 = 0
\]

\[
(x^{3/2} + 1)(x^{3/2} - 3) = 0
\]

The equation \( x^{3/2} + 1 = 0 \) has no solution since \( x^{3/2} \geq 0 \) for all \( x \). The equation \( x^{3/2} - 3 = 0 \) has the unique solution \( x = 3^{2/3} \) (which can also be written as \( x = 9^{1/3} \)). Hence the only solution to \( f'(x) = 5 \), and thus the only \( x \)-coordinate at which the tangent line is perpendicular to \( x + 5y = 1 \), is \( x = 9^{1/3} \).