(1) Use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ of the derivative to find $f'(x)$ when $f(x) = x^{-1/2}$.

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x} \sqrt{x+h}} \cdot \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{1}{\sqrt{x} \sqrt{x+h}} \cdot \frac{x - (x+h)}{\sqrt{x} + \sqrt{x+h}} \\
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{1}{\sqrt{x} \sqrt{x+h}} \cdot \frac{-h}{\sqrt{x} + \sqrt{x+h}} \\
= \lim_{h \to 0} \frac{1}{\sqrt{x} \sqrt{x+h}} \cdot \frac{1}{\sqrt{x} + \sqrt{x+h}} \\
= -\frac{1}{\sqrt{x} \sqrt{x} (2 \sqrt{x})} = -\frac{1}{2x^{3/2}} = -\frac{1}{2}x^{-3/2}
\]

(2) Use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ of the derivative to find $f'(x)$ when $f(x) = x^{-2}$.

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h^2} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \frac{x^2 - (x^2 + 2hx + h^2)}{x^2(x+h)^2} \\
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{-2hx - h^2}{x^2(x+h)^2} \\
= \lim_{h \to 0} \frac{-2x - h}{x^2(x+h)^2} \\
= \lim_{h \to 0} \frac{-2x}{x^2(x+h)^2} = -\frac{2x}{x^2} = -2x^{-3}
\]

(3) Assume $x$ is a number such that $\tan x = 7$ and $\sin x < 0$. Find $\sec x$.

We know $\sec^2 x = 1 + \tan^2 x = 1 + 7^2 = 50$. This implies $\sec x = \pm\sqrt{50} = \pm5\sqrt{2}$. Since $\frac{\sin x}{\cos x} = \tan x = 7 > 0$ and $\sin x < 0$, we conclude $\cos x < 0$, hence $\sec x = \frac{1}{\cos x} < 0$. This fact and $\sec x = \pm5\sqrt{2}$ imply $\sec x = -5\sqrt{2}$.

(4) Simplify $\sin(\sin^{-1} x)$, $\cos(\sin^{-1} x)$, $\sec(\sin^{-1} x)$, $\tan(\sin^{-1} x)$.
The definition of inverse function implies sin(sin\(^{-1}\) x) = x. Since
\[
\cos^2(\sin^{-1} x) = 1 - \sin^2(\sin^{-1} x) = 1 - (\sin(\sin^{-1} x))^2 = 1 - x^2,
\]
we conclude cos(\(\sin^{-1}\) x) = \(\pm\sqrt{1-x^2}\). But sin\(^{-1}\) x is in the interval \([-\pi/2, \pi/2]\) and cos is positive or zero on that interval. This implies cos(\(\sin^{-1}\) x) = \(\sqrt{1-x^2}\). Now we know
\[
\sec(\sin^{-1} x) = \frac{1}{\cos(\sin^{-1} x)} = \frac{1}{\sqrt{1-x^2}}.
\]
Finally,
\[
\tan(\sin^{-1} x) = \frac{\sin(\sin^{-1} x)}{\cos(\sin^{-1} x)} = \frac{x}{\sqrt{1-x^2}}.
\]

(5) Consider the function \(f(x) = \frac{x-8}{1+7x}\). Find a formula for the inverse function \(f^{-1}(x)\).

The notation \(t = f^{-1}(x)\) gives \(x = f(t) = \frac{t-8}{1+7t}\). The notation \(t = f^{-1}(x)\) gives \(x = f(t) = \frac{t-8}{1+7t}\). When we solve for \(t\), we get \(t-8 = x(1+7t)\), which is \(t-8 = x + 7xt\), which is \((1-7x)t = x + 8\), which is \(t = \frac{x + 8}{1-7x}\). We conclude \(f^{-1}(x) = t = \frac{x + 8}{1-7x}\).

(6) Solve for \(x\) in the equation \(e^{4x+3} = 2e^{3-x}\).

Dividing both sides of the equation \(e^{4x+3} = 2e^{3-x}\) by \(e^{3-x}\), we get \(e^{5x} = 2\). This is \(5x = \ln 2\), which is \(x = \frac{\ln 2}{5}\).

(7) Solve for \(x\) in the equation \(\sqrt{e^{8x-6}} = e^x\).

When we solve for \(t\), we get \(t-8 = x(1+7t)\), which is \(t-8 = x + 7xt\), which is \((1-7x)t = x + 8\), which is \(t = \frac{x + 8}{1-7x}\). We conclude \(f^{-1}(x) = t = \frac{x + 8}{1-7x}\).

(8) Solve for \(x\) in the equation \(e^{4x+3} = 2e^{3-x}\).

Dividing both sides of the equation \(e^{4x+3} = 2e^{3-x}\) by \(e^{3-x}\), we get \(e^{5x} = 2\). This is \(5x = \ln 2\), which is \(x = \frac{\ln 2}{5}\).

(9) Which of the given functions is even, which of the given functions is odd, and which of the given functions is neither? Explain carefully.

\[
f(x) = x^4\sqrt{1+x^2} \quad g(x) = x^3 + 1 \quad h(x) = x\sqrt{1+x^2}
\]

The function \(f(x)\) is even because
\[
f(-x) = (-x)^4\sqrt{1+(-x)^2} = x^4\sqrt{1+x^2} = f(x)
\]
The function $h(x)$ is odd because

$$h(-x) = -x \sqrt{1 + (-x)^2} = -(x \sqrt{1 + x^2}) = -h(x)$$

The function $g(x)$ is neither because

$$g(-x) = (-x)^3 + 1 = -x^3 + 1 \neq x^3 + 1 = g(x)$$
and

$$g(-x) = (-x)^3 + 1 = -x^3 + 1 \neq -(x^3 + 1) = -g(x)$$

(10) Find functions $f(x)$ and $g(x)$ such that $f(x)$ is even, $g(x)$ is odd and $f(x) + g(x) = 5x^5 - 7x^4 - 5x^3 + 8x^2 - x + 10$.

We have $f(x) = -7x^4 + 8x^2 + 10$ and $g(x) = 5x^5 - 5x^3 - x$. The function $f(x)$ is even because

$$f(-x) = -7(-x)^4 + 8(-x)^2 + 10 = -7x^4 + 8x^2 + 10 = f(x)$$

The function $g(x)$ is odd because

$$g(-x) = 5(-x)^5 - 5(-x)^3 - (-x) = -(5x^5 - 5x^3 - x) = -g(x)$$

(11) Express the function $f(x) = \sqrt{1 + \cos^2 x}$ as the composition of three simpler functions.

If $f_1(x) = \cos x$, $f_2(x) = 1 + x^2$ and $f_3(x) = \sqrt{x}$ then $f(x) = f_3(f_2(f_1(x)))$. 

3