Oral Exam Syllabus

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Algebraic Topology

[Hatcher]

- Fundamental group
  - Covering spaces, their classification, deck transformations
  - Seifert-van Kampen theorem
- Homology
  - Simplicial and singular homology
  - Exact sequences and excision, the Mayer-Vietoris sequence
  - Cellular homology and general axioms
- Cohomology
  - Universal coefficient theorem, Künneth formula
  - Cup product and the cohomology ring
  - Poincaré and Lefschetz duality
  - de Rham cohomology, de Rham's theorem

Riemann Surfaces

[Donaldson, Farkas - Kra]

- Examples of Riemann surfaces, properties of holomorphic maps
  - Quotients of group actions, smooth affine and projective curves, analytic continuation
  - Definition of holomorphicity, significance of meromorphic functions
  - Local behavior of holomorphic maps, degree, the Riemann-Hurwitz formula
- Calculus on Riemann surfaces
  - Differential forms on a real surface, Stokes' theorem
  - ∂ and $\bar{\partial}$ on a Riemann surface, Weyl's lemma
  - Meromorphic 1-forms, the residue theorem
- Riemann-Roch formula
- Riemann's bilinear relations, Abel-Jacobi map
- Poincaré-Koebe uniformization

References

[1] Simon Donaldson, *Riemann Surfaces*, Oxford University Press, 2011.

[2] Hershel M. Farkas, Irwin Kra, *Riemann Surfaces*, Springer, 1991.

[3] Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001.