

Syllabus for Oral Qualifying Exam

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Major Topics: Kirby Calculus and Knot Theory

Knot Theory

- Basic theory and standard families of examples
- Seifert surfaces, Knot factorization, and the Schönflies Theorem.
- The Kauffman bracket and Jones polynomial
- The Alexander invariants and the Conway polynomial
- Knot group, Fox calculus
- Goeritz matrix, cyclic branched covers, Arf invariant
- Slice Knots, Fox-Milnor Condition

Reference: An Introduction to Knot Theory by Lickorish (Chapters 1-12)

4 Manifolds and Kirby Calculus

- Intersection forms, signatures, Whiteheads theorem, statement of Freedman's theorem, conditions for existence of spin structures, almost complex structures, adjunction formula
- Surfaces in four manifolds, Complex surfaces, desingularization of curves,
- Handlebody decompositions, Morse theory
- Kirby diagrams, Kirby calculus, relative Kirby calculus
- Applications to 3-Manifolds (integral Dehn surgery)
- Plumblings, Spin structures, Branched Covers
- Cobordisms, h-cobordisms, and the h-cobordism theorem.

Reference: 4-Manifolds and Kirby Calculus by Gompf and Stipsicz (Chapters 1,2,4,5,6,9)

3-Manifolds

- Rational Dehn surgery
- Heegaard splittings
- Classification of genus 1 Heegaard diagrams (Lens spaces) and Seifert fibered spaces
- Even surgeries and characteristic sublink

Reference: Lectures on the Topology of 3-Manifolds by Saveliev (Chapters 1-9)

Minor Topics: Vector Bundles and Characteristic Classes

- Principle Bundle and vector bundle basics, classifying spaces
- Grassmanians and their cohomology rings
- Thom Isomorphism Theorem, Leray-Hirsh Theorem
- Classical Characteristic Classes: Stiefel-Whitney, Chern, Euler, and Pontryagin classes
- Unoriented cobordism ring
- Oriented cobordism ring (compute $\Omega_*^{SO} \otimes \mathbb{Q}$)
- Elementary obstruction theory
- Basics of K-Theory

References: Characteristic Classes by Milnor and Stasheff (Chapters 1-18) and Vector Bundles and K-Theory by Hatcher.