

# Oral Qualifying Exam Syllabus

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## 1 Major Topic: Conley Index Theory

- **Attractor-repeller pairs:** Definitions, Attractor-repeller pair decompositions, compatibility with Lyapunov functions.
- **Morse decompositions:** Definitions, compatibility with Lyapunov functions, Conley form, duality.
- **Conley’s decomposition theorem:** Chain recurrence, Conley’s decomposition theorem.
- **Conley index for flows:** Homotopy Conley index, independence of index pair, homology Conley index, continuation property, hyperbolic fixed point case, application of Thom isomorphism theorem.
- **Conley index for maps:** Index map, homotopy Conley index, shift-equivalence for choices of index pairs, homology Conley index, continuation property, hyperbolic fixed point case, compatibility with Morse index.
- **Connection matrices:** Connecting homomorphism, connection matrix for attractor-repeller decompositions, connection matrix for Morse decompositions, existence of connection matrix.
- **Transition matrices:** Basic definition, singular transition matrices, topological transition matrices.

## References

- [Sal85] Dietmar Salamon. “Connected Simple Systems and The Conley Index of Isolated Invariant Sets”. In: *Transactions of the American Mathematical Society* 291.1 (1985), pp. 1–41.
- [MM02] Konstantin Mischaikow and Marian Mrozek. “Conley Index”. In: *Handbook of Dynamical Systems*. Elsevier, 2002.

## 2 Minor Topic: K-Theory

- **Basic definitions for vector bundles:** Vector bundles, sections, morphisms.
- **Operations, sub-bundles, quotient bundles:** Direct sum, tensor, dual, Hom, and exterior power bundles. Definitions for sub-bundles, strict homomorphisms, Hermitian bundle.
- **Vector bundles on compact spaces:** Homotopy invariance, trivialization of bundles, existence of metrics on vector bundles, classifying bundle, homotopy theoretic definition for  $\text{Vect}_n(X)$ .
- **Definitions for K-Theory:** Grothendieck completion definition, homotopy-theoretic definitions for  $K(X)$ .
- **Elementary properties:** Relative K-theory definitions, exact sequence for relative K-theory, retracts, statement of Bott periodicity.
- **Computations of  $K^*(X)$ :**  $K^*(X)$  for cell complexes, projective bundles, etc.
- **Multiplication in  $K^*(X, Y)$ :** Definition for multiplication in  $K^*(X)$ ,  $K^*(X, Y)$  as a  $K(X)$ -module, Euler characteristic.
- **The Thom isomorphism:** Thom isomorphism, splitting principle.

## References

- [AA89] M.F. Atiyah and D.W. Anderson. *K-theory*. Advanced book classics. Basic Books, 1989. ISBN: 9780201407921. URL: <https://books.google.com/books?id=ZS9aAAAAYAAJ>.
- [Wei13] Charles A. Weibel. “The K-Book: An Introduction to Algebraic K-Theory”. In: 2013.