

Syllabus for the Oral Qualifying Exam

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1. Major Topic: Mathematical Analysis

1.1 Wave Equation

1. Fundamental solutions of linear wave equations
2. Energy estimates
3. Local existence for linear and nonlinear wave equations
4. Global existence using Klainerman-Sobolev inequality
5. Null conditions and global existence in 3+1 dimension
6. Strichartz estimates and their application
7. r^p -weighted energy estimates

1.2 Sobolev Spaces

1. Definition and basic properties
2. Approximation by smooth functions
3. Sobolev embedding theorem
4. Rellich-Kondrachov compact embedding theorem

1.3 Functional Analysis

1. Hahn-Banach theorem, Baire category theorem
2. Hilbert Space: Riesz Representation theorem, Lax-Milgram theorem and Hilbert Basis
3. L^p space: Interpolation, duality, completeness and compactness
4. Distribution: Definition, compactly supported distribution, tempered distribution, periodic distribution, convolution, Fourier transform

2. Minor Topic: Differential Geometry

2.1 Semi-Riemannian Geometry

1. Covariant Derivative and Parallel transport
2. Levi-Civita Connection
3. Geodesics and Tidal forces
4. Second Fundamental Form
5. Gauss-Codazzi Equation

2.2 Causality and General Relativity

1. Causal Structure
2. Domains of Dependence and Global Hyperbolicity
3. Singularity theorems and their proofs
4. Black holes, trapped surfaces
5. Schwarzschild black hole
6. Kerr black hole