Oral Qualifying Exam Syllabus

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Major Topic: Harmonic Analysis

- Interpolation Theory: Distribution functions, $L^{p,\infty}$ spaces and weak type operators, theorems of Marcinkiewicz and Riesz-Thorin, Young's convolution inequality
- Fourier Series: Riemann-Lebesgue lemma, Riemann localization principle, Dini and Jordan criteria for pointwise convergence, applications of the uniform boundedness principle, Dirichlet and Fejér kernels, L^p convergence for approximate identities, decay of Fourier coefficients, Gibbs phenomenon
- Fourier Transforms: Basic properties, Schwartz class and tempered distributions, Parseval and Plancherel theorems, Hausdorff-Young inequality for $1 \leq p \leq 2$, Poisson summation formula, translation invariant operators and multipliers, characterizations of $\mathscr{M}^{1,1}(\mathbb{R}^n)$ and $\mathscr{M}^{2,2}(\mathbb{R}^n)$, oscillatory integrals, sublevel set estimates and van der Corput's lemma
- The Hardy-Littlewood maximal function: Vitali Covering lemma, weak $(1,1)$ and strong (p,p) inequalities for $p > 1$, Lebesgue differentiation theorem and analogues, control of other maximal operators, strong maximal function, dyadic maximal function, Calderón-Zygmund decomposition, $L\log(L)$, TT^* method
- The Hilbert Transform: Poisson and conjugate kernels, weak $(1,1)$ and strong (p,p) theorems of Kolmogorov and M. Riesz, characterization of $Hf \in L^1$ for $f \in \mathcal{S}$, maximal function, truncated integrals and pointwise a.e. convergence, L^p convergence of Fourier series and integrals for $1 < p < \infty$ in one dimension, convergence in measure for $p = 1$
- Singular Integrals: Method of rotations, maximal function, Riesz transforms, Calderón-Zygmund theorem, Hörmander and gradient conditions, non-convolution type, vector-valued extension
- H^1 and BMO: Atoms, sharp maximal function, interpolation for $T : L^\infty \to BMO$, good-λ inequalities, duality, John-Nirenberg inequality
- Weighted Inequalities: A_p condition, reverse Hölder inequality, Hardy-Littlewood maximal function and singular integrals with respect to weights
- Littlewood-Paley Theory: Dyadic block decomposition, square function, Hörmander multiplier theorem, Marcinkiewicz multiplier theorem, spherical maximal function
- T1 Theorem: Cotlar's lemma, weak boundedness property

References:

- *Fourier Analysis* by Javier Duoandikoetxea; David Cruz-Uribe, SFO

- *Classical Fourier Analysis* by Loukas Grafakos

Minor Topics: The Hardy-Littlewood Circle Method and Ergodic Theory

- Weyl's Inequality: Dirichlet's approximation theorem, difference operators, fractional parts, Hua's lemma
- Waring's Problem: Generating function, major and minor arcs decomposition, singular integral, singular series
- Measure preserving systems: Poincaré recurrence theorem, ergodicity, associated unitary operators, von Neumann's mean ergodic theorem, Birkhoff's pointwise ergodic theorem, examples
- Continued Fractions: Gauss measure, continued fraction map, badly approximable numbers, Lagrange's theorem on quadratic irrationals
- Equidistribution: Weyl's criterion
- Calderón transference principle in relation to the Hardy-Littlewood maximal function

References:

- *Additive Number Theory, The Classical Bases* by Melvyn B. Nathanson, Chapters 4 and 5
- *Ergodic Theory With a View Towards Number Theory* by Manfred Einsiedler, Thomas Ward, Chapters 1-4