

Oral Qualifying Exam syllabus

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Major: Symplectic Geometry

1) Symplectic Manifolds

Symplectic Vector Spaces, Symplectic Structure on Manifold, Tautological Symplectic form on Cotangent Bundle, Lagrangian Submanifold, Symplectic and Hamiltonian Isotopies

2) Local theory of Symplectic Manifolds

Moser's Trick, Weinstein Lagrangian Neighborhood, Darboux Coordinates

3) Almost Complex Structure

Existence of compatible Almost Complex Structure, Complex manifolds, Kähler forms, basics of Dolbeault Theory.

4) Hamiltonian dynamics and moment map

Hamiltonian Vector fields, Hamiltonian actions, Moment maps, Marsden-Weinstein Reduction.

5) J -Holomorphic curves

Carleman similarity principle, Unique continuation, Critical points, Somewhere injective curves.

Reference

1) Ana Cannas da Silva. Lectures on Symplectic Geometry. Springer, Lectures notes in mathematics 1764, 2008.

2) Dusa McDuff and Dietmar Salamon. Introduction to Symplectic Topology. Oxford University Press.

3) Dusa McDuff, Dietmar Salamon. *J*– Holomorphic curves and Symplectic Topology.

Minor: Elliptic PDE

1) Laplace's Equation

Mean Value property, Max and min principle, harnack inequality, Green's Representation, Poisson integral, convergence theorems, Interior estimates of derivatives

2) Classical Maximum principle for linear Elliptic PDE

Weak Maximum principle, Hopf boundary point lemma and Strong Maximum principle.

3) Sobolev spaces

Weak derivatives, $W^{k,p}$ spaces, Regularization and approximation by smooth functions, Chain rule, Density theorems, Sobolev inequalities for $W_0^{1,p}$ space, Imbedding theorems for $W_0^{k,p}$ spaces, Potential estimates and Morrey's inequality, Kondrachov compactness for $W_0^{k,p}$ space.

4) Weak Solutions and elliptic regularity

Solvability of the Dirichlet Problem using Lax Milgram and Fredholm alternative, Differentiability of weak solutions, Global regularity.

Reference

1) David Gilbarg and Neil S. Trudinger. Elliptic Partial Differential Equations of Second Order.