

Oral Qualifying Exam Syllabus

1 Combinatorics

- **Enumerative:** Binomial coefficient identities, Inclusion-Exclusion, Stirling's Formula, Generating functions
- **Hypergraphs:** Sperner, Erdos-Ko Rado, Kruskal-Katona, Fisher's Inequality, Ray-Chaudhuri Wilson, Frankl-Wilson, Baranyai
- **Ramsey Theory:** Ramsey, Infinite Ramsey, Upper and Lower Bounds, Probabilistic lower bounds, Hales-Jewett and Van Der Waarden
- **Lattices:** Geometric and Distributive Lattices, Dilworth, Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin, Mobius inversion
- **Linear Algebra Method:** Combinatorial Nullstellensatz, Schwarz-Zippel Lemma
- **Extremal Problems:** Turan's Theorem, Statement of Regularity Lemma and its application to the Erdos-Stone Theorem, counting lemma, triangle removal lemma

2 Probabilistic Methods

- **Basics:** Linearity of Expectation, Markov's Inequality, Chernoff bounds, Alterations
- **Second Moment Method:** Chebyshev's Inequality, Threshold functions applications to properties of random graphs
- **Lovasz Local Lemma:** Symmetric and General forms, Applications to Ramsey Numbers
- **Poisson Paradigm:** Janson's Inequality, Brun's Sieve
- **Entropy:** Basic properties, Shearer's lemma, Brégman's Theorem
- **Martingales:** Definitions, Azuma's Inequality, Edge/Vertex Exposure

3 Graphs

- **Matchings:** Hall's Theorem, Tutte's Theorem
- **Planarity:** Euler's Theorem, Proof that K_5 and $K_{3,3}$ are not planar, Kuratowski, Wagner's Theorem

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- **Connectivity:** Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Matrix Tree Theorem, Cayley's Formula, Prufer Codes
- **Coloring:** Brooks' Theorem, Vizing's Theorem, 5 color theorem

4 Computational Complexity

- **Basics:** Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, independent set, 0/1 integer programming, conditions that imply $P \neq NP$
- **Decidability:** Diagonalization, Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem
- **Space Complexity:** definitions, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem, the Immerman-Szelepcsényi Theorem
- **Separation Theorems:** Time and Space Hierarchy Theorems (deterministic and nondeterministic versions)
- **Polynomial hierarchy:** Definitions of Σ_i, Π_i , complete problems, conditions that lead to the collapse of PH.
- **Circuits:** $P \subseteq P/poly$, CKT-SAT and alternate proof of Cook-Levin, Characterization of $P/poly$ as TMs with advice, Karp-Lipton Theorem, existence of hard functions, definitions of NC_i, AC_i
- **Probabilistic Complexity:** Definitions of RP, BPP and ZPP, error reduction, Sipser-Gacs Theorem, $BPP \subseteq P/poly$, randomized reductions and definition of BP, NP
- **Interactive Proofs** definitions, $dIP=NP$, $GNI \in AM$, $IP=PSPACE$