

Oral Qualifying Exam Syllabus

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1 Partial Differential Equations

1.1 Sobolev Spaces

1. Definition of Sobolev Spaces
2. Approximation by smooth functions
3. Extensions and Traces
4. Sobolev Inequalities : Gagliardo-Nirenberg Sobolev inequality, Morrey's Inequality
5. Compact Embedding Theorem : Rellich-Kondrachov Theorem
6. Poincaré's Inequality
7. Fourier Transform Method for $H^k(\mathbb{R}^n)$
8. The space H^{-1}
9. Spaces involving time

1.2 Second Order Elliptic Equations

1. Laplace Equation : Fundamental Solution, Mean Value Formula and Reciprocal, Smoothness, Local Estimates, Liouville Theorem
2. Definition of Weak Solutions
3. Existence by Lax-Milgram Theorem and Energy Estimates by Fredholm Alternative
4. Interior Regularity and Boundary Regularity
5. Weak and Strong Maximum Principles
6. Eigenvalues of symmetric elliptic operators

1.3 Second Order Parabolic Equation : the Heat Equation

1. Fundamental Solution
2. Mean Value Formula
3. Maximum Principle
4. Uniqueness and Backward Uniqueness by Energy method

1.4 Second Order Hyperbolic Equations

1. Wave equation : D'Alembert, Kirchhoff's, Poisson's Formulas, Uniqueness and Domain Dependence by Energy Methods
2. Helmholtz Equation : Green's Theorem and Formula in bounded and unbounded domains
3. Definition and Existence of Weak Solutions of Second Order Hyperbolic Equations

2 Functional Analysis

2.1 Banach Spaces

1. Metric Spaces, Normed Linear Vector Spaces, Banach spaces and Hahn-Banach Theorem
2. Baire Category Theorem, Uniform Boundedness Principle, Open Mapping Principle, Closed Graph Theorem, Banach Alaoglu Theorem
3. Duality, Reflexivity, Weak Topology, Weak* Topology

2.2 Hilbert Spaces

1. Definition of Hilbert Space and the Dual Space of Hilbert Space
2. Projection Lemma, Riesz Representation Theorem and Lax Milgram Theorem
3. Orthogonal Sets, Basis, Bessel's Inequality, Parseval's Theorem

2.3 Compact Operators

1. Bounded linear Operators
2. Definition, Elementary Properties, Adjoint in Banach Space and Hilbert Space
3. Riesz Fredholm Theory
4. The Spectrum of Compact Operators in Banach Spaces
5. The Spectrum of Compact Self Adjoint Operators in Hilbert Space
6. Hilbert Schmidt Theorem

References

- [Bre11] H Brezis. Functional analysis, sobolev spaces and partial differential equations, 2011.
- [CKK98] David L Colton, Rainer Kress, and Rainer Kress. *Inverse acoustic and electromagnetic scattering theory*, volume 93. Springer, 1998.
- [Eva22] Lawrence C Evans. *Partial differential equations*, volume 19. American Mathematical Society, 2022.