

Mathematics 495: Proofs from THE BOOK

Spring 2024

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Lectures: Monday and Thursday, 12:10 - 1:30 PM, SC 116, College Avenue Campus.

Office hours: Thursday, 3:50-5:10 pm, in Hill Center 340 or by appointment.

Course description: This is a buffet class of beautiful proofs from different areas of Mathematics. Paul Erdős often referred to "THE BOOK" in which God keeps the most elegant proof of each mathematical theorem. In this class we will study different proofs that were proposed as candidates for "THE BOOK".

Prerequisites: Mathematics 300 - Introduction to Mathematical Reasoning.

Textbook: *Proofs from THE BOOK* by Martin Aigner, Günter Ziegler, 6th edition, 2018.

Grades:

$$\text{Your Grade} = 0.4 \times \text{Homework} + 0.3 \times \text{Final} + 0.2 \times \text{Midterm} + 0.1 \times \text{Presentation}$$

Tentative list of topics:

- (1) Number Theory
 - (a) Six proofs of the infinity of primes
 - (b) Bertrand's postulate
 - (c) Binomial coefficients are (almost) never powers
 - (d) Representing numbers as sums of two squares
 - (e) The law of quadratic reciprocity
 - (f) Every finite division ring is a field
 - (g) The spectral theorem and Hadamard's determinant problem
 - (h) Some irrational numbers
 - (i) Four times $\pi^2/6$
- (2) Geometry
 - (a) Hilbert's third problem: decomposing polyhedra
 - (b) Lines in the plane and decompositions of graphs
 - (c) The slope problem
 - (d) Three applications of Euler's formula
 - (e) Cauchy's rigidity theorem
 - (f) The Borromean rings don't exist
 - (g) Touching simplices
 - (h) Every large point set has an obtuse angle
 - (i) Borsuk's conjecture
- (3) Analysis
 - (a) Sets, functions, and the continuum hypothesis
 - (b) In praise of inequalities
 - (c) The fundamental theorem of algebra
 - (d) One square and an odd number of triangles
 - (e) A theorem of Pólya on polynomials
 - (f) Van der Waerden's permanent conjecture
 - (g) On a lemma of Littlewood and Offord
 - (h) Cotangent and the Herglotz trick
 - (i) Buffon's needle problem
- (4) Combinatorics
 - (a) Pigeon-hole and double counting
 - (b) Tiling rectangles

- (c) Three famous theorems on finite sets
 - (d) Shuffling cards
 - (e) Lattice paths and determinants
 - (f) Cayley's formula for the number of trees
 - (g) Identities versus bijections
 - (h) The finite Kakeya problem
 - (i) Completing Latin squares
- (5) Graph Theory
- (a) Permanents and the power of entropy
 - (b) The Dinitz problem
 - (c) Five-coloring plane graphs
 - (d) How to guard a museum
 - (e) Turán's graph theorem
 - (f) Communicating without errors
 - (g) The chromatic number of Kneser graphs
 - (h) Of friends and politicians
 - (i) Probability makes counting (sometimes) easy

Attendance: Students are expected to attend all classes, but any student who has been told to quarantine, or is experiencing symptoms of any transmittable disease, should remain at home and not attend in-person class meetings. If you expect to miss class, please use the University absence reporting website <https://sims.rutgers.edu/ssra/> to indicate the date and reason for your absence. Please review the Rutgers policy on attendance.

Make-up Policy: Make-up Policy: Make-ups for midterm exams or other assessments are not guaranteed. Students must provide their instructor with a serious documented reason for why the assignment was missed to have the opportunity to make-up missed assignments. It is the student's responsibility to notify their instructor as soon as possible if emergency occurs that interferes with the student completing a scheduled assignment. If you experience a medical episode or family emergency before the end of the semester that will prevent you from properly demonstrating your academic accomplishment in the course, then you must arrange a T grade before the final exam. Once you take the final exam, the course work is complete, and you will be graded on that basis using the same standards that are applied to all other students.

Academic Integrity: Students are expected to maintain the highest level of academic integrity. You should be familiar with the university policy on academic integrity (<http://nbacademicintegrity.rutgers.edu/>). Violations will be reported and enforced according to this policy. Use of external website resources such as Chegg.com or others to obtain solutions to homework assignments, quizzes, or exams is cheating and a violation of the University Academic Integrity policy. Cheating in the course may result in grade penalties, disciplinary sanctions, or educational sanctions. Posting homework assignments, or exams, to external sites without the instructor's permission may be a violation of copyright and may constitute the facilitation of dishonesty, which may result in the same penalties as plain cheating. The Rutgers honor pledge will be included on all (major) assessments for you to sign: On my honor, I have neither received nor given any unauthorized assistance on this [examination/assignment].

Student-Wellness Services: Counseling, ADAP & Psychiatric Services (CAPS) (848) 932-7884 / 17 Senior Street, New Brunswick, NJ 08901. <http://health.rutgers.edu/medical-counseling-services/counseling/>

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The Office for Violence Prevention and Victim Assistance provides confidential crisis intervention, counseling, and advocacy for victims of sexual and relationship violence and stalking to students, staff and faculty. To reach staff during office hours when the university is open or to reach an advocate after hours, call 848-932-1181.

Disability Services: (848) 445-6800 / Lucy Stone Hall, Suite A145, Livingston Campus, 54 Joyce Kilmer Avenue, Piscataway, NJ 08854. <https://ods.rutgers.edu/>

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<https://ods.rutgers.edu/my-accommodations/letter-of-accommodations>

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: <https://ods.rutgers.edu/students/documentation-guidelines>. If the documentation supports your request for reasonable accommodations, your campus's disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss

the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: <https://ods.rutgers.edu/students/registration-form>.

Scarlet Listeners: (732) 214-9069 /
<https://rutgers.campuslabs.com/engage/organization/scarletlisteners>

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