Course Name: Mathematical Adventures in One-Dimensional Physics
Credits: 3 (two 80-minute meetings per week)
Pre-requisites: (244 and 250) or 252 or 292

Textbook: Carroll, Sean. *The Biggest Ideas in the Universe: space, time, and motion*. Dutton (2022).
Other Course Material: lecture notes distributed in class, journal articles and preprints on the world-wide web, YouTube videos of lectures by prominent scientists

Synopsis:

Imagine the universe consisted only of beads of different size and color on a single string, with some rules about how they would interact, combine, separate from, or bounce off one another. How would familiar notions of classical physics such as gravity or electromagnetism look like in such a universe? What about quantum phenomena? How far can one go on this quest? Is it feasible to look for a one-dimensional "theory of everything"?

This is the stuff of fundamental physics, and my goal in this course is to

- Introduce students to the mathematical tools they need in order to understand these questions
- Help them use those tools to find satisfactory answers to some of the questions
- Give them the vocabulary they need in order to ask many more such profound questions, not just about this cartoon world, but also about the actual world we live in.

The emphasis will be on mathematical rigor and exact analysis. Assignments include weekly readings, in-class discussions and presentations, a mid-term paper, and a final report.

During the first five weeks of the semester students will become familiar with elements of classical mechanics of particles and fields, the stationary action principle, and local conservation laws. The next six weeks are devoted to elements of quantum mechanics of particles, and the study of atomic Hamiltonians. The last two weeks of the semester will see us return to classical physics for a brief introduction to general relativity and gravitation.

Throughout the semester, students will learn how to do research in mathematical physics: They will learn how to look for reliable information on a topic, how to carry our bibliographic research, how to properly cite literature, and how to read with a critical eye. They will be given examples of original research done by other undergraduates at Rutgers, which they will be asked to read in detail, understand, make a brief oral presentation on, and write reviews in the style of referee reports about. They will practice their hand at coming up with their own mathematical conjectures, learn how to test their validity, and how to plot a course to prove or disprove those conjectures. They will submit a mock grant proposal to a fictitious funding agency, and participate in a review panel for the agency. For those who are inclined to do so, there will be an opportunity of collaborating on original, publishable research during a future semester, for example in the context of an independent research course.

Course Syllabus (tentative):

PART I

1. Preliminaries
    1. Physics, Mathematics, Philosophy
    2. Ontology, Epistemology, Nomology
    3. Space, Time, Space-time
    4. Particle, Field, Wave
    5. Position, Velocity, Acceleration
    6. Variable, function, functional
    7. The Stationary Action Principle
2. Classical Mechanics of Particles
    1. Lagrangian of point mechanics
    2. Momentum, Force, Equations of Motion
    3. Symmetry and Conservation
    4. Energy, Momentum, Energy-momentum
    5. Hamiltonian formulation of point mechanics
3. Mathematical Interlude I
    1. Functions of Several Variables
    2. ODEs and PDEs
    3. Initial value problems
    4. Transport equations
    5. Wave equations and how to solve them
    6. The Heaviside function and the Dirac delta function
4. Classical Mechanics of Fields
    1. Lagrangian density function
    2. Canonical velocities and momenta, Euler-Lagrange equations
    3. Symmetry and Conservation Laws, Noether’s Theorem
    4. Energy density-momentum density-stress tensor
5. Classical Mechanics of Particles and Fields Sourced by Them
    1. Massless scalar fields
    2. Particles as singularities of fields
    3. Goldilocks singularities
    4. Joint evolution of a field and its singularity
    5. The Agashe-Lee-T.Z. paper

PART II

6. Quantum Mechanics of Particles and Waves Guiding Them
    1. Physical space and Configuration space
    2. Wave function of a single particle and its evolution
    3. The guiding equation, particle trajectories
    4. Quantum probability current, Born's rule
7. Mathematical Interlude II
    1. Scalars, vectors, and linear transformations
    2. Matrices, Eigenvalues and Eigenvectors
    3. Tensor products and tensor spaces
    4. Hilbert spaces, self-adjoint operators and their spectrum
8. Quantum-mechanical Hamiltonians
    1. Bound states and scattering states
    2. Discrete and Continuous spectrum, energy eigenvalues
    3. The Born-Oppenheimer approximation
    4. One-dimensional hydrogen, the model
    5. The Dasgupta-Khurana-T.Z. paper
9. Interaction and Entanglement
    1. Wave function of a system of particles
    2. Conserved tensor currents
    3. Hypersurface Bohm-Dirac theory
    4. The guiding equation for a system of particles
    5. Nonlocality
10. Electrons and Photons
    1. Spin and Spinors
    2. Wave function of a single photon
    3. Electron-photon systems
    4. Compton scattering
    5. The Scanteianu-Wang paper
11. Gravity
    1. Spacetime geometry, geodesics, and curvature
    2. The equivalence principle
    3. General covariance
    4. Einstein's vacuum equations
    5. Einstein's non-vacuum equations
    6. Gravity in one space dimension
    7. The Boozer paper