Problem Set 6, Math 350, Fall 2017, Section 2

(1) (a) Let $T$ be a triangle in the plane whose vertices have all integer components. Show that the area of $T$ is at least $1/2$.

(b) Find a triangle in the plane whose vertices have all integer components and whose include $(0,0)$ and $(3,5)$, and whose area is $1/2$.

(2) Find the determinant of
\[
\begin{pmatrix}
  1 & 2 & 4 \\
  1 & 3 & 9 \\
  1 & 4 & 16
\end{pmatrix}
\]
using

(a) cofactor expansion along the second row.

(b) row-reduction.

(3) (a) More generally for any $x_1, x_2, x_3, y_1, y_2, y_3$ with $x_1, x_2, x_3$ distinct find the determinant of
\[
\begin{pmatrix}
  1 & x_1 & x_1^2 \\
  1 & x_2 & x_2^2 \\
  1 & x_3 & x_3^2
\end{pmatrix}
\]
using row reduction.

(b) Show that given three points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ with distinct $x_1, x_2, x_3$, there is a unique parabola $f(x) = a + bx + cx^2$ passing through the three given points, that is, with $f(x_k) = y_k$ for $k = 1, 2, 3$.

(3) (a) A permutation matrix is a matrix with exactly one $1$ in each row and column and the remaining entries all equal to $0$. Show that the determinant of any permutation matrix is $1$ or $-1$.

(b) An antisymmetric matrix is a matrix $A$ with the property that $A = -A^T$. Show that the determinant of any $5 \times 5$ skew-symmetric matrix is zero.

(4) (a) Let $u, v, w$ be vectors in $R^3$. Show that the matrix with columns $u - v, v - w, w - u$ has zero determinant.

(b) Suppose that the matrix whose columns are $u, v, w$ has determinant 2. What is the determinant of the matrix whose columns are $u + v, v + w, w + u$?