Suggested Syllabus for Math 502 (revised after 2014 offering)

Functions of Bounded Variation (if not covered in 501)

1. Functions of Bounded Variation, definitions and basic properties.
2. Jordan decomposition of BV functions.
3. BV functions have only countably many jump discontinuities.

Differentiation

4. Uniform absolute continuity of the integral
5. Simple Vitali Lemma
6. Lebesgue Differentiation theorem.
7. Hardy-Littlewood maximal function and weak (1,1) estimate; derivation of Lebesgue differentiation theorem from this fundamental estimate.
8. Points of density of a set.
9. Vitali covering theorem
10. Consequence of (9), BV functions are differentiable almost everywhere.
11. Cantor function
12. Total variation function $V(x)$ and $V'(x) = |f'(x)|$ for BV functions.
13. Absolutely continuous and singular functions.

9-13 is the buildup to the **Fundamental theorem of Calculus**.

14. The Fundamental theorem of Calculus and absolute continuity.
15. Basic properties of Convex functions, differentiation, Lipschitz continuity etc.
16. Jensen's inequality.

L^p Spaces

17. Definitions of L^p, $1 \leq p \leq \infty$.
18. Hölder and Minkowski inequality and integral version of Minkowski inequality.
19. Banach and metric space properties of L^p spaces like completeness and separability of L^p spaces.
20. Continuity of the L^p norm $||f(x+h) - f(x)||_p \to 0$ as $h \to 0$.

Convolutions and Approximation to the Identity

21. Convolution, basic properties, Young's convolution inequality.
22. Approximate identities, convergence in L^p and pointwise, convergence of approximations to the identity, this is to be viewed as further application of the Hardy-Littlewood maximal function.
23. Density of $C_0^\infty(R^n)$ in $L^p(R^n)$ spaces when $1 \leq p < \infty$, applications of 21-22.
24. Some examples of approximate identities, heat kernel, Poisson kernel.
25. The Fourier transform, properties of the Fourier transform on $L^1(R^n)$ and $L^2(R^n)$ for example Riemann-Lebesgue lemma,Parseval and Plancherel theorem— example of another application of approximate identities.

Hilbert Spaces

26. Basic Hilbert spaces, definition, parallelogram theorem.
27. Subspace theorem for Hilbert spaces. $X = W \oplus W^\perp$ for W a closed subspace of a Hilbert space X.
28. Orthonormal basis for Hilbert spaces, Complete orthonormal systems.
29. Riesz-Fischer theorem for Hilbert spaces.
30. Riesz representation theorem for Hilbert spaces.

Supplementary material, Fourier Series

31. Basic Fourier series on $(-\pi, \pi)$, Riemann-Lebesgue lemma.

32. Effect of smoothness of a function on Fourier coefficients, Holder continuity and its effect, for example.

33. Fejer's theorem for Fourier series as an example of an application of Approximate identities to Fourier series.

34. Parseval and Plancherel formula as an application of orthonormal systems of Hilbert spaces as applied to Fourier series.

35. Bernstein's theorem and absolutely convergent Fourier series, application of 32,34.

36. Hardy's Tauberian theorem coupled with Fejer's theorem yields any Lipschitz function has an almost everywhere convergent Fourier series.